Brief Announcement: Superpeer formation amidst churn and rewiring

Bivas Mitra, Sujoy Ghose, Niloy Ganguly
Department of Computer Science and Engineering, IIT Kharagpur, India

1 Introduction

In this paper, we develop an analytical framework to explain the appearance of bimodal degree distribution in popular superpeer networks like Gnutella, KaZaA, where a large number of low degree peer nodes are connected with small number of high degree, resourceful superpeer nodes [3]. The emergence of superpeer networks is driven by (a) bootstrapping protocol which attaches incoming nodes towards 'resourceful' peers, (b) peer churn and (c) link rewiring. We use rate equations and build up a formalism encompassing the three processes which helps us to understand the emergence of superpeer networks. We model bootstrapping protocols through node attachment rules where probability of attachment of the incoming peer to an online node is proportional to the degree of the online node. We realistically assume that bandwidth of a node is finite which restricts its maximum connectivity (*cutoff degree*). A node j, after reaching its cutoff degree $k_c(j)$, rejects any further connection requests from the incoming peers. During churn, the departing node disconnects all the connections with its neighboring nodes. During rewiring, a node disconnects itself from any one of its neighboring nodes and preferentially reconnects to a high degree node in the network.

2 Development of growth model in face of peer churn and link rewiring

In this section, we intend to compute the degree distribution p_k in face of joining and removal of nodes in addition to the rewiring of links. We assume (simplistically) that all nodes have the same cutoff degree k_c (hence same bandwidth). The p_k can be computed by observing the shift in the number of k (and $k-1$) degree nodes to $k+1$ (and k) degree nodes at each timestep t. Since rewiring does not change the total number of nodes and links in the network and addition & removal of the node is performed with probability q and r respectively, therefore, the total number of k degree nodes at timestep $t+1$ becomes $(n+q-r)p_{k,t+1}$. Hence, assuming $p_{k,t+1} = p_{k,t} = p_k$ in asymptotic case [1], the change in the number of k degree nodes between the timesteps t and $t+1$ becomes

$$\Delta n_k = (n+q-r)p_{k,t+1} - np_{k,t} = (q-r)p_k \tag{1}$$

Joining of a node: The probability that an online peer of degree k will receive a new link from the incoming peer is given by

$$A_k \;=\; kp_k\Big(\sum_{k_1=0}^{k_c-1} k_1 p_{k_1}\Big)^{-1} = kp_k(zf)^{-1}, \;\; k < k_c; \;\; 0 \;\; otherwise \tag{2}$$

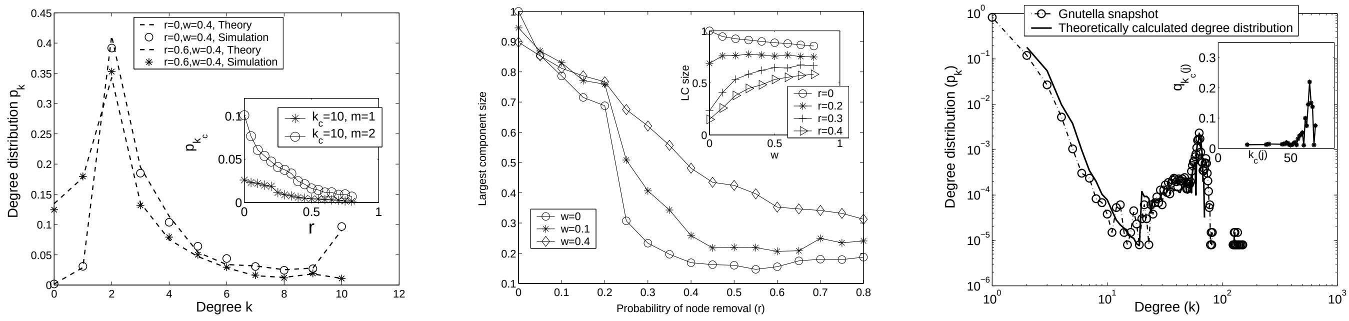
where $f = \left(1 - \frac{k_c p_{k_c}}{z}\right)$ is a parameter and $\sum_{k=0}^{k_c} kp_k = z$ is the average degree of the network.
We assume that each new node joins with degree m, hence the mean number of nodes of degree k that gains an edge from the incoming node and moves to degree $k+1$ can be expressed as $\delta^{jo}_{k\to(k+1)} = m \times A_k = m\frac{kp_k}{zf}$. Therefore, the net change in the number of k degree nodes due to joining of a new node

$$\delta^{jo}_k = \delta^{jo}_{(k-1)\to k} - \delta^{jo}_{k\to(k+1)} = m\left((k-1)p_{k-1} - kp_k\right)(zf)^{-1} \tag{3}$$

Removal of a node: Removal of a node affects the number of k degree nodes in three different ways; (a) itself removal of a k degree node (b) reduction in the number of k degree nodes due to the removal of their neighboring nodes (c) similarly increase in the number of k degree nodes due to removal of nodes which are neighbors of $k+1$ degree nodes. Hence average number of k degree nodes that lose one link and become a node of degree $k-1$ is $\delta^{rm}_{k\to(k-1)} = \sum_{j=0}^{k_c} \frac{jp_j kp_k}{\langle k\rangle} = kp_k$. Therefore, the net change in the number of k degree nodes due to node removal

$$\delta^{rm}_k = (-p_k + \delta^{rm}_{(k+1)\to k} - \delta^{rm}_{k\to(k-1)}) = (-p_k + (k+1)p_{k+1} - kp_k) = (k+1)[p_{k+1} - p_k] \tag{4}$$

Rewiring of a link: The rewiring leads to change in the number of nodes of degree k in two different ways; (a) link disconnection and (b) link reconnection. Due to the random disconnection of the old link, a fraction of $(k+1)$ (and k) degree nodes lose one link and move in to degree k (and $(k-1)$). The probability of landing at a k degree node following a randomly chosen link (that is going to be disconnected) is $\frac{kp_k}{z}$.



(a) Degree distribution of the emerging network in face of peer churn and link rewiring where $k_c = 10$ and $m = 2$. Inset shows that churn reduces p_{k_c}.

(b) The change in the LCC with respect to churn rate r for different rewiring probability w. Inset shows the change in the LCC with respect to the w.

(c) Comparative study between the real world Gnutella network and our theoretical model. The inset shows the cutoff degree distribution $q_{k_c(j)}$.

Figure 1: Various impacts resulting from bootstrapping, churn and rewiring.

Hence mean reduction in the k degree to $(k-1)$ degree nodes becomes $\delta^{dis}_{k\to(k-1)} = \frac{kp_k}{z}$. Similarly, during reconnection the mean number of k degree nodes that (preferentially) accepts a new link and moves from degree k to $k+1$ becomes $\delta_{k\to(k+1)} = \frac{kp_k}{zf}$. Hence

$$\delta^{dis}_k = \delta^{dis}_{k\to(k-1)} - \delta^{dis}_{(k+1)\to k} = (kp_k - (k+1)p_{k+1})z^{-1} \tag{5}$$

$$\delta^{recon}_k = \delta^{recon}_{(k-1)\to k} - \delta^{recon}_{k\to k+1} = ((k-1)p_{k-1} - kp_k)(zf)^{-1} \tag{6}$$

$$\delta^{relink}_k = (\delta^{recon}_k - \delta^{dis}_k) \tag{7}$$

Using Eqs. (3)–(7) we can write rate equations to formulate the change in the number of k degree nodes in the network which at each timestep attaches a new node of degree m with probability q, removes a node with probability r and rewires links with probability w. Four pertinent degree ranges $k = 0$, $k = m$, $k \neq 0, m, k_c$ and $k = k_c$ need to be taken into consideration.

$$\Delta n_k = (q-r)p_k = \begin{cases} q\delta^{jo}_k + r\delta^{rm}_k + w\delta^{relink}_k; & 0 < k < k_c,\ k \neq m \\ q(1+\delta^{jo}_m) + r\delta^{rm}_m + w\delta^{relink}_m; & k = m \\ q\delta^{jo}_{(k-1)\to k} + r(-p_k - \delta^{rm}_{k\to(k-1)}) + w(\delta^{recon}_{(k-1)\to k} - \delta^{dis}_{k\to(k-1)}); & k = k_c \\ r(-p_k + \delta^{rm}_{(k+1)\to k}) + w\delta^{dis}_{(k+1)\to k}; & k = 0 \end{cases} \tag{8}$$

The rate equations for $k = m, k_c$ and 0 are special cases for the following reasons. For $k = m$, beyond the normal increase, the entrance of the node itself with degree m adds an additional member in the m-degree node family. Since the nodes having degree k_c are not allowed to take any incoming link, nodes only accumulate at degree $k = k_c$. Nodes having degree $k = 0$ do not lose any link. The degree distribution p_k of the emerging networks can be found by recursively solving the Eq. (8).

Validation

In order to validate theory, we consider two different (q, w, r) cases; (a) (1.0, 0.0, 0.4) (b) (1.0, 0.6, 0.4). We simulate a network with 5000 nodes following the rules of bootstrapping, churn and rewiring and perform 500 individual realizations. Fig. 1(a) shows that the agreement between the theoretical (Eq. (8)) and simulation results is exact in terms of average degree distribution which validates the correctness of the theoretical model (dashed lines show theoretical results whereas symbols depict simulation results).

3 Impact of peer churn and rewiring

In this section, we investigate the influence of peer churn and link rewiring on the topological properties like (a) superpeer fraction and (b) the largest connected component (LCC). In both the cases, initially we examine the impact of churn in absence of rewiring. Next, we include rewiring in our analysis to understand their combined effect.

Impact on superpeer fraction

Peer churn: Fig. 1(a) shows that in the absence of peer churn, a spike appears at around degree k_c which means the accumulation of superpeer nodes in the network. In theory also, substituting $r = w = 0$ in Eq. (8), we find $p_{k_c} > p_{k_c-1}$ since $k_c >> 1$. However, we find that the increase in r results in a sharp fall in

p_{k_c}. As churn probability r gets higher than the threshold r_c, the spike at $k = k_c$ disappears ($p_{k_c} \leq p_{k_c-1}$). Therefore, using Eq. (8) (substituting $w = 0$ for $k = k_c$), we find

$$p_{k_c} = (qm(k_c - 1)(zf)^{-1})(q + rk_c)^{-1}p_{k_c-1} \Rightarrow r_c \geq q(m(zf)^{-1} - k_c{}^{-1}) \tag{9}$$

From the above expression, it becomes directly evident that higher k_c and m make the spike more robust.
Rewiring: We find that assuming $r = 0$ and $0 < w < 1$, Eq. (8) results $p_{k_c} > p_{k_c-1}$ which confirms that rewiring preserves bimodality in the absence of churn. However, in face of churn, the spike disappears (i.e. $p_{k_c} \leq p_{k_c-1}$) if the threshold churn

$$r_c > (q(m(k_c - 1) - zf) + w((k_c - 1) - fk_c))(k_c zf)^{-1} \tag{10}$$

Interestingly, rewiring (w) lowers the value of r_c if $(k_c - 1) - fk_c < 0 \Rightarrow z > k_c^2 p_{k_c}$. Hence, for a given k_c, the value of p_{k_c} needs to be above some threshold to make rewiring useful.

Impact on largest connected component (LCC)

Peer churn: Churn reduces the amount of superpeer nodes in the network and in effect weakens the connectivity among the nodes within the LCC. The dissolution of the largest component happens due to the sudden percolation of 'holes' in the networks at $r \approx 0.2$ which disintegrates the network into a large number of *small disconnected components*. Interestingly, r remains largely independent of network size.
Rewiring: *Moderate rewiring gives benefit.* In presence of proper rewiring, p2p network shows graceful degradation in face of churn; the nodes largely remain connected, however, the diameter of the network increases. Rewiring produces 'bridging links' between the 'moderate size' components (Fig 1(b)). *Heavy rewiring is not cost effective, sometimes detrimental.* Inset of Fig. 1(b) indicates that if the churn rate is lower than some threshold value ($r < 0.073$), rewiring itself may be detrimental as disconnection of links removes smaller components from the network. On the other extreme, beyond a threshold level, the impact of rewiring saturates and further increase does not improve the network connectivity.

4 Multiple cutoff degrees and Gnutella network

In reality, nodes join the network with various bandwidth connections like dial up, ISDN, ADSL, leased line etc. Subsequently, the cutoff degrees of individual nodes become different from one another. Similar to [2], we derive the degree distribution for the case where $q_{k_c(j)}$ fraction of nodes join with cutoff degree $k_c(j)$ (inset of Fig. 1(c)). The derivation is not shown here, however it can almost accurately replicate the degree distribution of Gnutella[1] (Fig. 1(c)). We describe the evolution of Gnutella network due to joining, removal of nodes and rewiring of links by the tuple (q, r, w, m). We set $q = 1.0$ and $m = 2$. To obtain r and w, we fit the calculated degree distribution with Gnutella snapshot, obtaining an excellent match at $r = 0.474$ (47.4% churn) and $w = 0.249$ (24.9% nodes rewire). This theoretical result is reinforced by the measurement study of [4] on the dynamics of Gnutella network which also reports heavy churn.

5 Conclusion

The development of this analytical framework actually facilitates in discovering various non-intuitive facts, helps us to understand the intricate relationship existing among various parameters (churn, rewiring, p_{k_c} etc.), which network engineers can exploit to design better robust and efficient p2p system. The best part, however lies in almost accurately mimicking the complex degree distribution of Gnutella network.

References

[1] A. Barabási and R. Albert. Emergence of scaling in random networks. *Science*, 286(5439):509–512, October 1999.

[2] B. Mitra, A. K. Dubey, S. Ghose, and N. Ganguly. How do superpeer networks emerge? In *INFOCOM 2010*. IEEE, March 2010.

[3] Y. J. Pyun and D. S. Reeves. Constructing a balanced, log(n)-diameter super-peer topology. In *IEEE P2P*. Zurich, Switzerland, August 2004.

[4] D. Stutzbach, R. Rejaie, and S. Sen. Characterizing unstructured overlay topologies in modern p2p file-sharing systems. *IEEE/ACM Trans. Netw.*, 16(2):267–280, 2008.

[1] We simulate Gnutella network following the snapshot obtained from the Multimedia & Internetworking Research Group, University of Oregon, USA (http://mirage.cs.uoregon.edu/P2P/info.cgi). The snapshot is collected by the research group during September 2004 and the size of the network simulated from the snapshot is of 1,31,869 nodes.