Foundations of Computing Science (CS60001)
Class Test 2 Solution

2^{nd} November, 2011

1. (a) i. $RR \Leftrightarrow R : C_1$
   ii. $BB \Leftrightarrow \neg B : C_2$
   iii. $GG \Leftrightarrow \neg R : C_3$
   iv. $(R \wedge \neg B \wedge \neg G) \vee (\neg R \wedge B \wedge \neg G) \vee (\neg R \wedge \neg B \wedge G) : C_4$
   v. $(\neg RR \wedge \neg BB \wedge \neg GG) \vee (\neg RR \wedge \neg BB \wedge GG) \vee (\neg RR \wedge BB \wedge \neg GG) \vee (RR \wedge \neg BB \wedge \neg GG) : C_5$

   (b) Let us assume:
   $X = (R \wedge \neg B \wedge \neg G)$
   $Y = (\neg R \wedge B \wedge \neg G)$
   $Z = (\neg R \wedge \neg B \wedge G)$
   Therefore $C_4 = X \vee Y \vee Z$

   i. From X and C_2, we can infer BB
   From Y and C_3, we can infer GG
   From Z and C_1, we can infer $\neg RR$
   So, from C_4, C_2, C_3, C_1, we get
   $(\neg RR \vee BB \vee GG)$ which is equivalent to $\neg(RR \wedge \neg BB \wedge \neg GG)$

   ii. From X and C_1, we can infer RR
   From Y and C_2, we can infer $\neg BB$
   From Z and C_3, we can infer GG
   So, from C_4, C_1, C_2, C_3, we get
   $(RR \vee \neg BB \vee GG)$ which is equivalent to $\neg(\neg RR \wedge BB \wedge \neg GG)$

   iii. From X and C_1, we can infer RR
   From Y and C_3, we can infer GG
   From Z and C_2, we can infer BB
   So, from C_4, C_1, C_3, C_2, we get
   $(RR \vee BB \vee GG)$ which is equivalent to $\neg(\neg RR \wedge \neg BB \wedge \neg GG)$

   iv. From our last 3 inferences and C_5, we get $(\neg RR \wedge \neg BB \wedge GG)$

   (c) From 1(b)iv, we infer $\neg BB$. From $\neg BB$ and C_2, we get B. i.e., the treasure is in the blue chest.

2. $C_1 : \forall x(Lion(x) \Rightarrow Fierce(x))$
   $C_2 : \exists x(Lion(x) \wedge \neg Drinks\_Coffee(x))$

   (a) False. Consider a domain D, where, $\forall x(\neg Lion(x) \wedge \neg Fierce(x))$ is true. Since there are no lions, C_1 is vacously true. But there is no x for which $Fierce(x)$ is true, so there are no fierce creatures.

   (b) True. Let $G = \exists x Fierce(x)$.
   Rewriting C_1,C_2 and $\neg G$ in clause form, we get:
   $S_1 : \neg Lion(x) \vee Fierce(x)$
   $S_2 : Lion(a)$
   $S_3 : \neg Drinks\_Coffee(a)$
   $S_4 : \neg Fierce(x)$

From S_1 and S_2 we get $Fierce(a)$ (Let this be S_5).
From S_4 and S_5 we get $False$.

(c) True.
Let $G = \exists x\{Fierce(x) \land \neg Drinks\_Coffee(x)\}$.
Rewriting C_1,C_2 and $\neg G$ in clause form, we get:
$S_1 : \neg Lion(x) \lor Fierce(x)$
$S_2 : Lion(a)$
$S_3 : \neg Drinks\_Coffee(a)$
$S_4 : \neg Fierce(x) \lor Drinks\_Coffee(x)$.
From S_1 and S_2 we get $Fierce(a)$ (Let this be S_5).
From S_5 and S_4 we get $Drinks\_Coffee(a)$ (Let this be S_6).
From S_3 and S_6 we get $False$.

3. (a) $\forall x Loves(x, Jerry)$

(b) $\forall x \exists y Loves(x, y)$

(c) $\exists x \forall y Loves(y, x)$

(d) $\neg \exists x \forall y Loves(x, y)$

(e) $\exists x \neg Loves(Tom, x)$

(f) $\exists x \forall y \neg Loves(y, x)$

(g) $\exists x[\{\forall y Loves(y, x)\} \land \forall z\{x \neq z \Rightarrow \exists y \neg Loves(y, z)\}]$

(h) $\exists x \exists y[(x \neq y) \land Loves(Jerry, x) \land Loves(Jerry, y) \land \forall z\{(x \neq z \land y \neq z) \Rightarrow \neg Loves(Jerry, z)\}]$

(i) $\forall x Loves(x, x)$

(j) $\exists x[Loves(x, x) \land \forall y\{(x \neq y) \Rightarrow \neg Loves(x, y)\}]$