Counter-Example Guided Abstraction Refinement

Testing & Verification
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This presentation is based on the work of E. Clarke, A. Gupta, J. Kukula, O. Strichman (CAV’02). Most of these slides are from A. Gupta’s presentation.
Model Checking

- **Given:**
  - Finite transition system $M(S, I, R, L)$
  - A temporal property $p$

- **The model checking problem:**
  - Does $M$ satisfy $p$?

\[ M \models p \]
Temporal properties:

- “Always x=y”
  \( (G(x=y)) \)
- “Every Send is followed immediately by Ack”
  \( (G(Send \rightarrow X Ack)) \)
- “Reset can always be reached”
  \( (GF \text{ Reset}) \)
- “From some point on, always switch_on”
  \( (FG \text{ switch_on}) \)

“Safety” properties

“Liveness” properties
Model Checking (safety)

Add reachable states until reaching a fixed-point

\[ \text{\( \textcolor{red}{\text{bad state}} \)} \]
Model Checking (safety)

Too many states to handle!

= bad state
Abstraction

Abstraction Function $h : S \rightarrow S'$
Abstraction Function

- **Partition variables into visible(V) and invisible(I) variables.**

- The abstract model consists of V variables. I variables are made inputs.

- The abstraction function maps each state to its projection over V.
Abstraction Function

Group concrete states with identical visible part to a single abstract state.
Existential Abstraction
Model Checking Abstract Model

- Preservation Theorem

\[ M' \models p \rightarrow M \models p \]

- Converse does not hold

\[ M' \not\models p \not\leftrightarrow M \not\models p \]

- The counterexample may be spurious
Checking the Counterexample

- **Counterexample**: \((c_1, \ldots, c_m)\)
  - Each \(c_i\) is an assignment to \(V\).

- **Simulate the counterexample on the concrete model.**
Checking the Counterexample

Concrete traces corresponding to the counterexample:

\[ \phi \quad = \quad I(s_1) \quad \wedge \quad (\text{Initial State}) \]

\[ \bigwedge_{i=1}^{m-1} R(s_i, s_{i+1}) \quad \wedge \quad (\text{Unrolled Transition Relation}) \]

\[ \bigwedge_{i=1}^{m} \text{visible}(s_i) = c_i \quad (\text{Restriction of V to Counterexample}) \]
Abstraction-Refinement Loop

- M, p, h
  Abstract
  M', p
  Model Check
  Pass
  No Bug

  h'
  Refine

  Spurious
  Check
  Counterexample
  Fail
  Real
  Bug

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**Refinement**

**Abstraction/refinement with conflict analysis**

(Chauhan, Clarke, Kukula, Sapra, Veith, Wang, FMCAD 2002)

- Simulate counterexample on concrete model with SAT
- If the instance is unsatisfiable, analyze conflict
- Make visible one of the variables in the clauses that lead to the conflict
Why spurious counterexample?

Deadend states

Bad States

Failure State
Refinement

- Problem: Deadend and Bad States are in the same abstract state.
- Solution: Refine abstraction function.
- The sets of Deadend and Bad states should be separated into different abstract states.
Refinement

Refinement: $h'$
Deadend States

\[ \phi_D = I(s_1) \land \bigwedge_{i=1}^{f-1} R(s_i, s_{i+1}) \land \bigwedge_{i=1}^{f} \text{visible}(s_i) = c_i \]
Refinement

\[ \phi_B = R(s_f, s_{f+1}) \land \]
\[ \text{visible}(s_f) = c_f \land \text{visible}(s_{f+1}) = c_{f+1} \]
Refinement as Separation

<table>
<thead>
<tr>
<th>$d_1$</th>
<th>0 1 0 0 1 0 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>$b_1$</td>
<td>0 1 0 0 0 1 0</td>
</tr>
<tr>
<td>$b_2$</td>
<td>0 1 0 0 1 1 1</td>
</tr>
</tbody>
</table>

Refinement: Find subset $U$ of $I$ that separates between all pairs of deadend and bad states. Make them visible.

Keep $U$ small!
Refinement as Separation

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Keep $U$ small!
Refinement as Separation

The state separation problem

Input: Sets D, B

Output: Minimal \( U \in I \) s.t.:

\[ \forall d \in D, \forall b \in B, \exists u \in U. \quad d(u) \neq b(u) \]

The refinement \( h' \) is obtained by adding \( U \) to \( V \).
Two separation methods

- **ILP-based separation**
  - Minimal separating set.
  - Computationally expensive.

- **Decision Tree Learning based separation.**
  - Not optimal.
  - Polynomial.
Separation with Decision Tree Learning (Example)

Classification:

\[ d_1 = (0, 1, 0, 1) \]
\[ d_2 = (1, 1, 1, 0) \]
\[ b_1 = (1, 1, 1, 1) \]
\[ b_2 = (0, 0, 0, 1) \]

Separating Set:
\[ \{v_1, v_2, v_4\} \]
Separation with 0-1 ILP (Example)

\[ d_1 = (0, 1, 0, 1) \quad b_1 = (1, 1, 1, 1) \]
\[ d_2 = (1, 1, 1, 0) \quad b_2 = (0, 0, 0, 1) \]

\[ \text{Min} \quad \sum_{i=1}^{4} v_i \]

subject to:

\[ v_1 + v_3 \geq 1 \quad /* \text{Separating } d_1 \text{ from } b_1 */ \]
\[ v_2 \geq 1 \quad /* \text{Separating } d_1 \text{ from } b_2 */ \]
\[ v_4 \geq 1 \quad /* \text{Separating } d_2 \text{ from } b_1 */ \]
\[ v_1 + v_2 + v_3 + v_4 \geq 1 \quad /* \text{Separating } d_2 \text{ from } b_2 */ \]
Separation with 0-1 ILP

Min $\sum_{i=1}^{\vert I \vert} v_i$

subject to: $(\forall d \in D) \ (\forall b \in B) \ \sum_{1 \leq i \leq \vert I \vert, \ d, b \ \text{differ at} \ v_i} v_i \geq 1$

- One constraint per pair of states.
- $v_i = 1$ iff $v_i$ is in the separating set.
Refinement as Learning

- For systems of realistic size
  - Not possible to generate $D$ and $B$.
  - Expensive to separate $D$ and $B$.

- Solution:
  - Sample $D$ and $B$
  - Infer separating variables from the samples.

- The method is still complete:
  - Counterexample will eventually be eliminated.
The CMU CEGAR Tool

- NuSMV
- Chaff
- LpSolve
- Dec Tree
- Cadence SMV
- MC
- Sep
- SAT